Hayden Middle School
Mathematics Tournament 2019
Algebra I

1) It is necessary to have a 40% antifreeze solution in the radiator of a certain car. The radiator now has 50 liters of 20% solution. How many liters of this should be drained and replaced with 100% antifreeze to get the desired strength? 1) _____

A) 12.5 L B) 16.7 L C) 25 L D) 20 L

Find the partial fraction decomposition for the rational expression.

2) $\frac{13x - 34}{x^2 - 6x + 8}$ 2) _____

A) $\frac{9}{x + 4} + \frac{4}{x + 2}$ B) $\frac{4}{x - 4} + \frac{9}{x - 2}$ C) $\frac{1}{x - 4} + \frac{1}{x - 2}$ D) $\frac{9}{x - 4} + \frac{4}{x - 2}$

3) Given that $f(x) = e^{x - 1} + 3$, find $f^{-1}(x)$ and give the domain and range of $f^{-1}(x)$. 3) _____

A) $f^{-1}(x) = \ln(x - 3) + 1$, domain = $(0, \infty)$, range = $(0, \infty)$

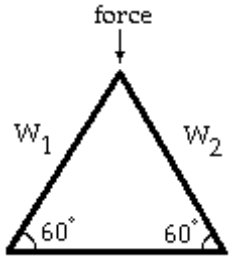
B) $f^{-1}(x) = \ln(x - 3) + 1$, domain = $(3, \infty)$, range = $(-\infty, \infty)$

C) $f^{-1}(x) = \ln(x - 1) + 3$, domain = $(3, \infty)$, range = $(-\infty, \infty)$

D) $f^{-1}(x) = \ln(x - 1) + 3$, domain = $(-\infty, \infty)$, range = $(-\infty, \infty)$

Solve the system to find W_1 and W_2.

4) Linear systems occur in the design of roof trusses for new homes and buildings. The simplest type of roof truss is a triangle. The truss shown in the figure is used to frame roofs of small buildings. If a force of 116 pounds is applied at the peak of the truss, then the forces or weights W_1 and W_2 exerted parallel to each rafter of the truss are determined by the following linear system of equations. 4) _____



$\frac{\sqrt{3}}{2}(W_1 + W_2) = 116$

$W_1 - W_2 = 0$

A) $W_1 = -66.97$ lb; $W_2 = -66.97$ lb B) $W_1 = 66.97$ lb; $W_2 = 66.97$ lb

C) $W_1 = 66.97$ lb; $W_2 = 58$ lb D) $W_1 = 38.67$ lb; $W_2 = 38.67$ lb

Solve

5) $x^3 - 64 = 0$ 5) _____

A) $\{4, 2 \pm 2\sqrt{5}\}$ B) $\{4, -2 \pm 2i\sqrt{3}\}$ C) $\{4, 2 \pm 2i\sqrt{6}\}$ D) $\{4, -2 \pm 2i\}$

Evaluate the determinant.

6) $\begin{vmatrix} 5 & 4 & 6 \\ 3 & 7 & 3 \\ 4 & 4 & 9 \end{vmatrix}$ 6) _____

A) 771 B) 3 C) 99 D) -99

7) The sum of a student's three scores is 214. If the first is 21 points more than the second, and the sum of the first two is 25 more than twice the third, what was the first score? 7) _____

A) 42 points B) 65 points C) 86 points D) 63 points

Solve the equation for the indicated variable. Assume no denominator is 0.

8) $2x^2 - 4xy + 3y^2 = 1$, for y 8) _____

A) $y = \frac{2x \pm 2\sqrt{3 - 2x^2}}{3}$ B) $y = \frac{2x \pm \sqrt{6 - 4x^2}}{3}$

C) $y = \frac{2x \pm 4\sqrt{3 - 2x^2}}{3}$ D) $y = \frac{2x \pm \sqrt{3 - 2x^2}}{3}$

9) A lumber yard has fixed costs of $2590.80 per day and variable costs of $0.34 per board-foot produced. Lumber sells for $1.54 per board-foot. How many board-feet must be produced and sold daily to break even? 9) _____

A) 7620 board-feet B) 2159 board-feet

C) 1439 board-feet D) 1378 board-feet

10) A real estate development consists of home sites that range in width from 53 to 105 feet and in depth from 125 to 187 feet. Using x as the variable in both cases, write absolute value inequalities that correspond to these ranges. 10) _____

A) $|B - 52| \le 53$, $|x - 62| \le 125$ B) $|B - 53| \le 52$, $|x - 125| \le 62$

C) $|x - 79| \le 26$, $|x - 156| \le 31$ D) $|x - 26| \le 79$, $|x - 31| \le 156$

11) A flare fired from the bottom of a gorge is visible only when the flare is above the rim. If it is fired with an initial velocity of 208 ft/sec, and the gorge is 672 ft deep, during what interval can the flare be seen? ($h = -16t^2 + v_0t + h_0$.) 11) _____

A) $6 < t < 7$ B) $0 < t < 6$ C) $12 < t < 13$ D) $18 < t < 19$

12) A company makes 3 types of cable. Cable A requires 3 black, 3 white, and 2 red wires. B requires 1 black, 2 white, and 1 red. C requires 2 black, 1 white, and 2 red. The company used 100 black, 110 white and 90 red wires. How many of each type of cable were made? 12) _____

A) 10 A; 103 B; 20 C B) 20 A; 30 B; 10 C

C) 10 A; 30 B; 20 C D) 10 A; 30 B; 93 C

13) A pendulum bob swings 6.0 cm on its first oscillation. On each subsequent oscillation the bob travels $\frac{1}{3}$ of the previous distance. Find the total distance the bob travels before coming to rest. 13) _____

A) 9.0 cm B) 13.0 cm C) 13.3 cm D) 12.0 cm

14) How many different 4-letter radio-station call letters can be made if the first letter must be K or W, repeats are allowed, but the call letters cannot end in an O? 14) _____

A) 35,152 B) 16,900 C) 33,800 D) 456,976

Write the binomial expansion of the expression.

15) $\left(\frac{1}{x} - \sqrt{13}\,y\right)^3$ 15) _____

A) $\frac{1}{x^3} - \frac{3\sqrt{13}\,y}{x^2} + \frac{39y^2}{x} - 13\sqrt{13}\,y^3$

B) $\frac{1}{x^3} + \frac{3\sqrt{13}\,y}{x^2} + \frac{39y^2}{x} + 13\sqrt{13}\,y^3$

C) $\frac{1}{x^3} - \frac{3\sqrt{13}\,y}{x^2} - \frac{39y^2}{x} - 13\sqrt{13}\,y^3$

D) $\frac{1}{x^3} - \frac{3\sqrt{13}\,y}{x^2} + \frac{117y^2}{x} - 13\sqrt{13}\,y^3$

Solve

16) $(2x + 3)^{1/2} + (4 - x)^{1/2} = 4$ 16) _____

A) $\varnothing$ B) {3} C) {-3} D) $\left\{3, \frac{11}{9}\right\}$

17) Find the sum of the first 144 positive even integers. 17) _____

A) 20,736 B) 20,880 C) 20,592 D) 21,025

18) The stadium vending company finds that sales of hot dogs average 48,000 hot dogs per game when the hot dogs sell for $2.50 each. For each 50 cent increase in the price, the sales per game drop by 5000 hot dogs. What price per hot dog should the vending company charge to realize the maximum revenue? 18) _____

A) $4.80 B) $3.65 C) $3.90 D) $2.30

Find all complex zeros of the polynomial function. Give exact values. List multiple zeros as necessary.

19) $f(x) = 3x^4 - 10x^3 + 20x^2 - 40x + 32$ 19) _____

A) $\frac{4}{3}, 2, -i, i$ B) $\frac{4}{3}, 2, -2i, 2i$ C) $-\frac{4}{3}, -2, -2i, 2i$ D) $\frac{2}{3}, 2, -4i, 4i$

20) Find the required annual interest rate, to the nearest tenth of a percent, for $1100 to grow to $1400 if interest is compounded monthly for 6 years. 20) _____

A) 4.0% B) 2.7% C) 8.1% D) 0.3%

Solve the problem. Round to the nearest tenth unless indicated otherwise.

21) The resistance of a wire varies directly as the length of the wire and inversely as the square of the diameter of the wire. A 20 foot length of wire with a diameter of 0.1 inch has a resistance of 3 ohms. What would the resistance be for a 21 foot length, with diameter 0.01 inch, of the same kind of wire ? 21) _____

A) 308 ohms B) 312.5 ohms C) 315 ohms D) 327 ohms

Determine the function value.

22) Suppose $f(x) = \log_a(x)$ and $f(2) = 2$. Find $f\left(\frac{\sqrt{2}}{2}\right)$ 22) _____

A) 1 B) -2 C) 2 D) -1

Perform the indicated operations and simplify.

23) $\frac{8}{2m^2 - 9mp - 5p^2} + \frac{5}{8m^2 + 2mp - p^2} - \frac{2}{4m^2 - 21mp + 5p^2}$ 23) _____

A) $\frac{41m - 35p}{(2m + p)(m - 5p)(4m - p)}$

B) $\frac{33m - 31p}{(2m + p)(m - 5p)(4m - p)}$

C) $\frac{33m - 35p}{(2m + p)(m - 5p)(4m - p)}$

D) $\frac{41m - 31p}{(2m + p)(m - 5p)(4m - p)}$

24) Cal is packing his suitcase to go on a trip. He wants to pack 3 pairs of pants chosen from the 8 pairs of pants in his closet and 4 shirts chosen from the 10 shirts in his closet. In how many ways can this be done? 24) _____

A) 282,240 B) 70,560 C) 1,693,440 D) 11,760

25) Use the properties of exponents to write the function of the form $f(t) = ka^t$, where k is a constant. 25) _____

$f(t) = \left(\frac{1}{2}\right)^{3 - 4t}$

A) $f(t) = \left(\frac{1}{16}\right)8^t$

B) $f(t) = \left(\frac{1}{8}\right)16^t$

C) $f(t) = 8 \cdot 16^t$

D) $f(t) = \frac{1}{8}\left(\frac{1}{16}\right)^t$

Answer Key
Testname: HAYDEN MIDDLE ALGEBRA I 2019

1) A
2) D
3) B
4) B
5) B
6) C
7) C
8) D
9) B
10) C
11) A
12) C
13) D
14) C
15) A
16) D
17) B
18) B
19) B
20) A
21) C
22) D
23) C
24) D
25) B